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The inverse estimation of the thermal boundary behavior of a heated cylinder normal to a laminar air stream

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Abstract

This study provides an inverse analysis to determine the thermal boundary behavior of a heated cylinder normal to a laminar air stream. A finite-difference method is used to discretize the governing equations and then a linear inverse model is constructed. The present approach is to rearrange the matrix forms of the governing differential equations and to estimate the unknown conditions of the cylinder from the linear inverse model. The linear least-squares error method is then adopted to find the solutions of the unknown thermal boundary conditions such as surface temperatures, local heat flux, local Nusselt numbers, and even the unknown temperature of the hot wire imbedded in the center of the cylinder. The results show that only few measuring points inside the cylinder are needed to estimate the unknown quantities of the thermal boundary behavior even when measurement errors are considered. From this study it is confirmed that the proposed method is effective and applicable for the two-dimensional inverse heat conduction problems. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

Recently, the inverse analysis has been widely applied to many design and manufacturing problems especially when direct measurements of surface conditions are not possible. We may not obtain precise results when the data is difficult to measure. However, by solving the inverse problem, we can obtain precise results only with numerical computations and simple instruments. All inverse problems are ill-posed, and a small measurement error will induce a large estimated error [1–5]. For example, it would be difficult to measure the temperatures or the heat flux at the tool-work interface in a machining operation, inside a combustion chamber, at the outer surface of a re-entry vehicle, and on the irregular surface. In all these cases, the inverse analysis for the heat conduction and convection problems can be successfully used to deal with the determination of the crucial boundary thermal parameters, such as heat transfer coefficients, Nusselt numbers, surface heat flux, internal energy sources, contact conductance and thermal properties.

The estimation for the boundary conditions in the inverse heat conduction problems has received a great attention in the recent years [6–8]. Various methods, analytical or numerical, have been developed to solve the inverse heat conduction and convection problems. Traditionally, the inverse problem includes two phases: the process of analysis and the process of optimization. In the analysis process, the unknown conditions are assumed and then the results of the problem are solved directly through the numerical methods such as finite-difference

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Nomenclature

$\mathbf{A}$	constant matrix constructed from thermal properties and space coordinates
$\mathbf{B}$	coefficient matrix of $\mathbf{C}$
$\mathbf{C}$	vector constructed from the unknown boundary conditions
$\mathbf{D}$	matrix constructed from the functions of the boundary conditions
$\mathbf{E}$	product of $\mathbf{A}^{-1}$ and $\mathbf{B}$
$\mathbf{F}$	error function
$h(\theta)$	function of the unknown local heat transfer coefficients
k	thermal conductivity
$Nu(\theta)$	function of the unknown local Nusselt numbers
$q(\theta)$	function of the unknown local heat flux
$\mathbf{R}$	reverse matrix
R	radius of the cylinder
(r, θ)	cylindrical coordinate
T	temperature
$T(\theta)$	function of the unknown surface temperature
$T(r, \theta)$	temperature at each grid point (r, θ)
$\mathbf{T}$	temperature matrix
U	velocity

Greek symbols

Δr	increment of radial coordinate
$\Delta\theta$	increment of angular coordinate
ω	random error of the measurement
σ	bound of the quantity ω

Subscripts

c	cylinder
estimated	estimated data
exact	exact data
i	index of radial coordinate
j	index of angular coordinate
measured	measured data
s	surface
w	hot wire
∞	uniform air stream

methods and finite-element methods. The solutions from the above process are used to integrate with data measured at the interior point of the solid. Thus, a nonlinear problem is constructed for the optimization process. In the optimization process, an optimizer, such as the conjugated gradient method, the steepest decent method and so on, is used to guide the exploring points systematically to search for a new set of guess conditions, which is then substituted for the unknown conditions in the analysis process.

This work uses a methodology of the reverse matrix method [9,10] to solve the inverse problems. This method rearranges the matrix forms of the governing differential equations in order to represent the unknown conditions explicitly. The linear least-squares error method is then adopted to find the solutions for the unknown boundary conditions. Additionally, no explicit functional form is assumed for the boundary conditions where as in other methods polynomial or series forms are employed.

Because of the complexity of the flow around a cylinder, many researchers in classical fluid mechanics have taken this problem as their research topics. Coutanceau and Bouard [11,12] proposed the experimental determination of the main features of the steady and unsteady viscous flow in the wake of a circular cylinder in uniform translation. Lin and Pepper [13] employed a numerical method to investigate the separated flow around a circular cylinder.

In many circumstances, we must deal with the problems concerning the heat transfer on the surface of heated objects with the flow around them. For instance, the design of the heat exchanger, the application of a hot wire anemometer and the thermal analysis of the turbine blade subjected to a high temperature flow. In these cases, the influence of the temperature distribution on the surface of objects should be carefully studied. Giedt [14] investigated experimentally the variation of point unit-heat-transfer coefficient around a cylinder normal to an air stream at high Reynolds numbers. Jain and Goel [15] presented a numerical study for unsteady laminar forced convection from a circular cylinder at low Reynolds numbers. Without discussing the thermal behavior inside the cylinder, all of the previously mentioned literatures only discussed the properties of the flow around the cylinder and the thermal behavior on the cylindrical surface. Tseng [16] developed an algorithm called the direct sensitivity coefficient (DSC) method to deal with the IHCP in an annular cylinder.

In this paper, a methodology to solve the inverse problem about a heated cylinder normal to a laminar air stream is presented. The unknown thermal boundary conditions such as surface temperatures, local heat flux, local Nusselt numbers, and even the unknown temperature of the hot wire will be obtained simultaneously through this proposed inverse method. In contrast to the previous studies, the advantages of this method are that no prior information is needed on the functional form of the unknown quantities, that no initial guesses are required and that the iterations in the calculating process can be avoided. Furthermore, the uniqueness of the solutions can easily be identified.

2. Physical model

Consider the problem of a cylinder with surface temperature, $T(\theta)$ placed in a uniform air stream of temperature, T_∞ and velocity, U_∞. According to the symmetric characteristics, only a half domain of the cylinder is considered, as shown in Fig. 1. The cylinder is considered to be long enough so that the end effects can be neglected and accordingly the problem is assumed two-dimensional. For simplification, the hot wire imbedded in the center of the heated cylinder is assumed to be a point source and is maintained at constant temperature, T_w. In addition, the effect of temperature variation on fluid properties is assumed negligible and the fluid is incompressible.

The governing equation for the temperature field of the cylinder can be expressed as

$$\frac{\partial^2 T(r,\theta)}{\partial r^2}+\frac{1}{r}\frac{\partial T(r,\theta)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T(r,\theta)}{\partial \theta^2}=0$$

$$0 \le r \le 0.05\,(\mathrm{m})$$

$$0 \le \theta \le \pi\,(\mathrm{rad}) \tag{1}$$

where $T(r,\theta)$ is the temperature at each grid point (r,θ).

The appropriate boundary conditions are

$$\frac{\partial T(r,\theta)}{\partial \theta}=0 \quad \theta=0 \tag{2}$$

$$\frac{\partial T(r,\theta)}{\partial \theta}=0 \quad \theta=\pi \tag{3}$$

$$T(0,\theta)=T_w \quad r=0 \tag{4}$$

$$q(\theta)=-k_c\frac{\partial T(r,\theta)}{\partial r}=h(\theta)\left[T(0.05,\theta)-T_\infty\right] \quad r=0.05\,\mathrm{m} \tag{5}$$

where $q(\theta)$ is the local heat flux at the interface between the cylinder and the flow around the cylinder, k_c is the thermal conductivity of the cylinder, $h(\theta)$ is the local heat transfer coefficient, and T_∞ is the temperature of the uniform air stream.

The following constant parameters are used: the radius of the cylinder $R = 0.05$ m, the thermal conductivity of the cylinder $k_c = 14$ W/m K (nichrome), the external air temperature $T_\infty = 300$ K, and the thermal conductivity of the air stream $k_\infty = 0.0263$ W/m K. Moreover, the hot wire temperature is $T_w = 473$ K, which is used to heat the cylinder.

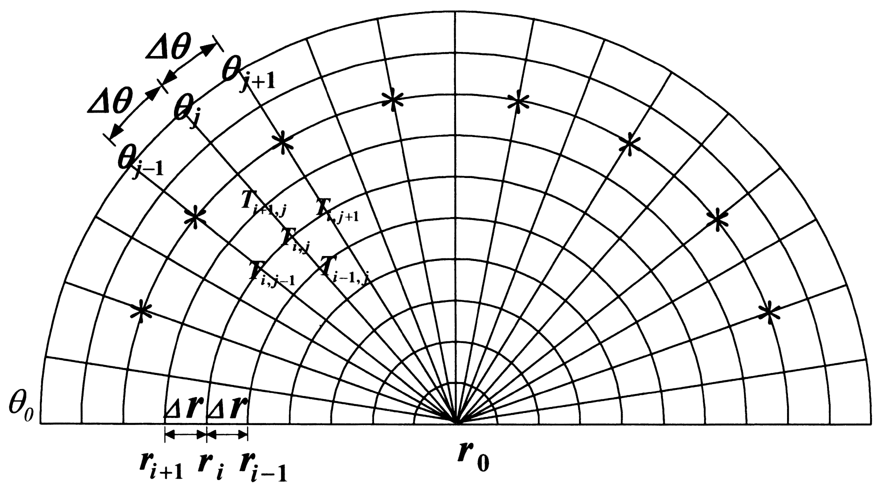

Fig. 1. The cylindrical coordinate system with measurement locations ∗.

3. Numerical method

3.1. The direct problem

The finite-difference method is employed in the analysis process. After discretization, the governing equation, Eq. (1), and the boundary condition, Eq. (5), can be expressed in the following recursive forms:

$$\frac{1}{(\Delta r)^2}\left(T_{i+1,j}-2T_{i,j}+T_{i-1,j}\right)+\frac{1}{r_i}\frac{1}{2\Delta r}\left(T_{i+1,j}-T_{i-1,j}\right)+\frac{1}{r_i^2}\frac{1}{(\Delta\theta)^2}\left(T_{i,j+1}-2T_{i,j}+T_{i,j-1}\right)=0 \tag{6}$$

$$q_{s,j}=-k_c\frac{T_{s,j}-T_{s-1,j}}{\Delta r}=h_{s,j}\left(T_{s,j}-T_\infty\right) \tag{7}$$

where $\Delta r = 0.005$ m and $\Delta\theta = \pi/18$ are the increments in the spatial coordinates, $T_{i,j}$ is the temperature at the grid (i, j), the subscript i is the ith grid along the r coordinate, the subscript j is the jth grid along the θ coordinate, and the subscript s represents the grid on the surface or boundary.

In regard to the treatment of the boundary conditions, Eq. (7), the segments used on the boundary are as many as there are nodes. Thus, the values of the heat transfer coefficients $h_{s,j}$ at different nodes on the boundary are treated as distinct.

Using the recursive forms an equivalent matrix equation can be expressed as

$$\mathbf{AT}=\mathbf{D} \tag{8}$$

where matrix $\mathbf{A}$ is a constant matrix, which is constructed from the thermal properties and the spatial coordinates. The components of matrix $\mathbf{T}$ are the temperatures at discretized points, and the components of matrix $\mathbf{D}$ are the function of the boundary conditions. The direct analysis is to determine the temperatures at the nodes when all the boundary conditions and thermal properties are known. The direct problem expressed in Eq. (8) can then be solved using the Gauss elimination method.

In this study, we use the temperature data obtained from the direct problem to simulate the measured temperatures of the interior points of the cylinder in the inverse problem, and the boundary conditions required to solve the direct problem are given in the work by Yang et al. [17].

3.2. The inverse problem

For the inverse problem, matrix $\mathbf{A}$ can be constructed according to the known physical model and numerical methods, and matrix $\mathbf{T}$ is composed of the temperatures inside the cylinder measured by the thermocouples. Decoupling the coefficients of the components of matrix $\mathbf{D}$ will transform the direct formulation to the following inverse forms:

$$\mathbf{AT}=\mathbf{BC} \tag{9}$$

where $\mathbf{D}=\mathbf{BC}$, $\mathbf{B}$ is the coefficient matrix of $\mathbf{C}$, and $\mathbf{C}$ is the vector of the unknown boundary conditions, such as the temperature of the hot wire, the Nusselt numbers, heat flux, temperatures and heat transfer coefficients of the discretized points on the cylindrical surface.

Matrix $\mathbf{C}$ of the constructed inverse model, Eq. (9), can then be solved by the linear least-squares error method. Assuming that the estimated data of $\mathbf{C}_{\text{estimated}}$ can be obtained by means of the given estimated temperature $\mathbf{T}_{\text{estimated}}$, i.e.,

$$\mathbf{AT}_{\text{estimated}}=\mathbf{BC}_{\text{estimated}} \tag{10}$$

$$\mathbf{T}_{\text{estimated}}=\mathbf{A}^{-1}\mathbf{BC}_{\text{estimated}}=\mathbf{EC}_{\text{estimated}} \tag{11}$$

where $\mathbf{E}=\mathbf{A}^{-1}\mathbf{B}$.

Comparing the estimated data $\mathbf{T}_{\text{estimated}}$ with the measured data $\mathbf{T}_{\text{measured}}$, the error function $\mathbf{F}$ can be represented as

$$\mathbf{F}=(\mathbf{T}_{\text{estimated}}-\mathbf{T}_{\text{measured}})^{\mathrm{T}}(\mathbf{T}_{\text{estimated}}-\mathbf{T}_{\text{measured}}) \tag{12}$$

Substituting Eq. (11) into Eq. (12), we can express $\mathbf{F}$ as the following matrix equation

$$\begin{aligned}\mathbf{F}&=(\mathbf{EC}_{\text{estimated}}-\mathbf{T}_{\text{measured}})^{\mathrm{T}}(\mathbf{EC}_{\text{estimated}}-\mathbf{T}_{\text{measured}})\\&=\left(\mathbf{C}^{\mathrm{T}}_{\text{estimated}}\mathbf{E}^{\mathrm{T}}-\mathbf{T}^{\mathrm{T}}_{\text{measured}}\right)(\mathbf{EC}_{\text{estimated}}-\mathbf{T}_{\text{measured}})\\&=\mathbf{C}^{\mathrm{T}}_{\text{estimated}}\mathbf{E}^{\mathrm{T}}\mathbf{EC}_{\text{estimated}}-\mathbf{T}^{\mathrm{T}}_{\text{measured}}\mathbf{EC}_{\text{estimated}}\\&\quad-\mathbf{C}^{\mathrm{T}}_{\text{estimated}}\mathbf{E}^{\mathrm{T}}\mathbf{T}_{\text{measured}}+\mathbf{T}^{\mathrm{T}}_{\text{measured}}\mathbf{T}_{\text{measured}}\end{aligned} \tag{13}$$

We can minimize $\mathbf{F}$ by differentiating $\mathbf{F}$ with respect to $\mathbf{C}_{\text{estimated}}$ as

$$\frac{\partial\mathbf{F}}{\partial\mathbf{C}_{\text{estimated}}}=0 \tag{14}$$

From Eq. (14), we obtain

$$\mathbf{E}^{\mathrm{T}}\mathbf{EC}_{\text{estimated}}+\mathbf{E}^{\mathrm{T}}\mathbf{EC}_{\text{estimated}}-\mathbf{E}^{\mathrm{T}}\mathbf{T}_{\text{measured}}-\mathbf{E}^{\mathrm{T}}\mathbf{T}_{\text{measured}}=0$$

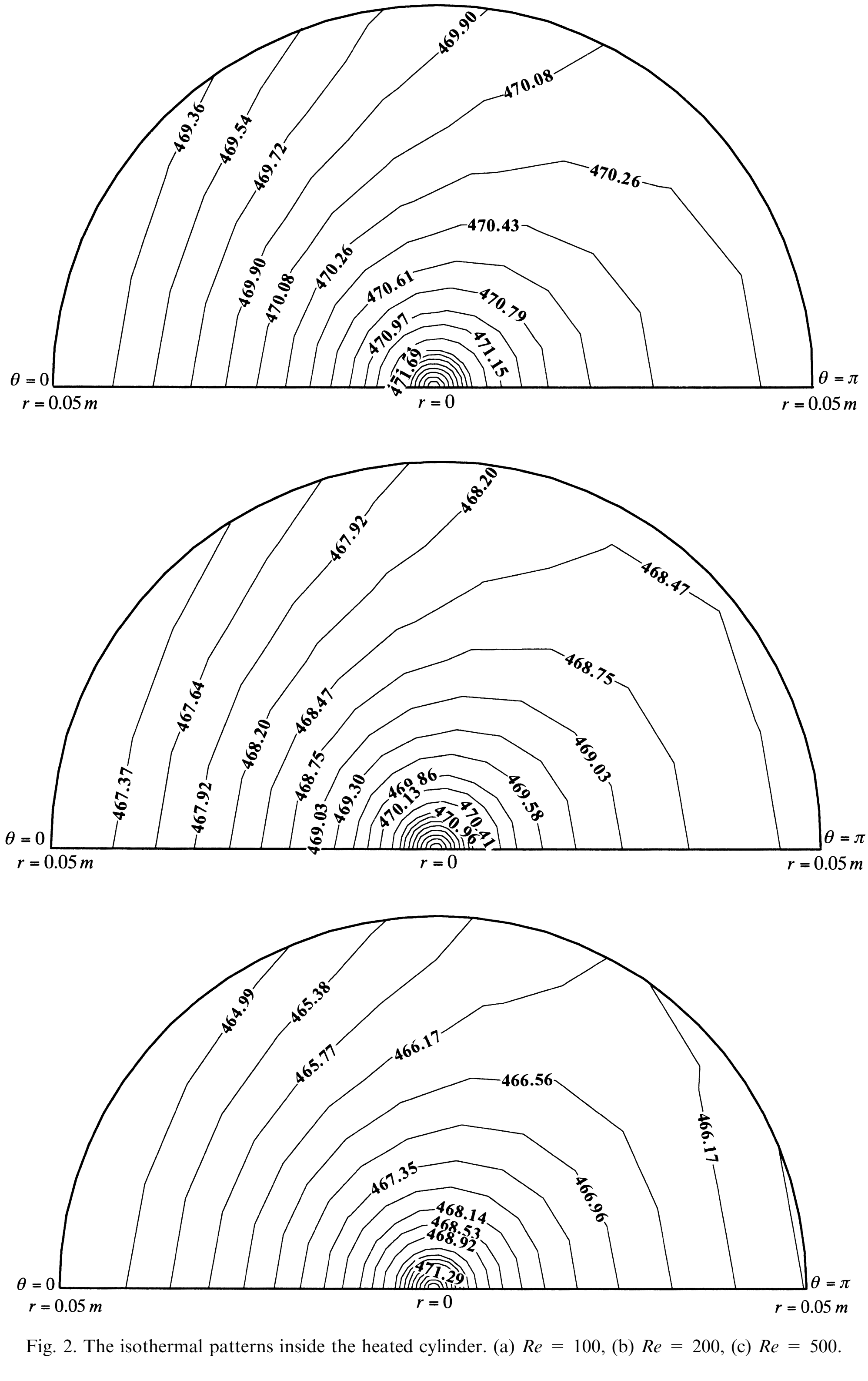

Fig. 2. The isothermal patterns inside the heated cylinder. (a) $Re = 100$, (b) $Re = 200$, (c) $Re = 500$.

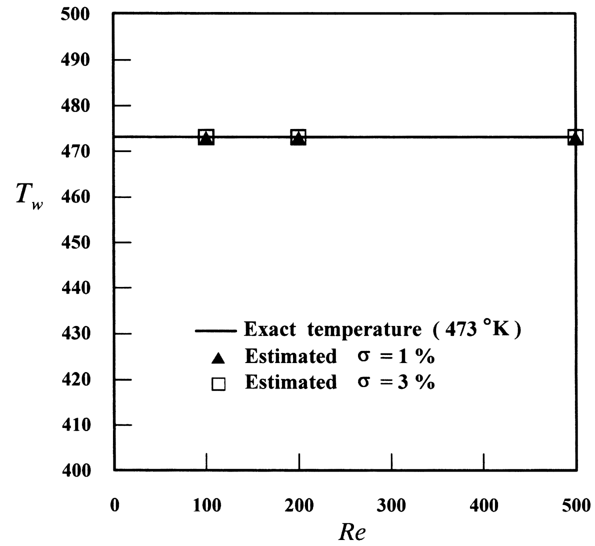

Fig. 3. The hot wire temperature predicted by the proposed inverse method. ($\sigma = 0$, 1 and 3%).

$$\mathbf{E}^{\mathrm{T}}\mathbf{E}\mathbf{C}_{\mathrm{estimated}} = \mathbf{E}^{\mathrm{T}}\mathbf{T}_{\mathrm{measured}} \tag{15}$$

Thus, matrix $\mathbf{C}_{\mathrm{estimated}}$ can then be solved as follows:

$$\mathbf{C}_{\mathrm{estimated}} = (\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}\mathbf{T}_{\mathrm{measured}} \tag{16}$$

where, $(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}$ is the reverse matrix of the inverse problem and is denoted by **R**. The process derived above is the linear least-squares error method [18].

Eq. (16) is formulated to represent the measurements of all the discretized points in the problems. In most cases, not all of the points need to be measured. The realistic experimental approach is to measure only few points in the problems. Therefore, only parts of matrix **R**, **T** and vector **C** corresponding to the measuring positions need to be constructed in order to estimate the boundary conditions of the inverse problems. In general, when a large portion of the matrices and vector are selected, i.e., when the number of transducers or measuring points are large, the cost of computation and experiment increase; the accuracy of the estimated results increases as well.

Estimating matrix **C**, we can obtain simultaneously the temperatures of the surface and the hot wire, the local heat transfer coefficients $h(\theta)$, the local Nusselt numbers $Nu(\theta)$ and the local heat flux $q(\theta)$ on the cylindrical surface. In addition, a special feature of this approach is that the iteration in the calculating process can be avoided and the problem can be solved in a linear domain.

In the inverse problem, it is important to investigate the stability of the estimation. Usually, a minor measurement error makes the estimation away from the exact solution in the ill-posed inverse problem. The

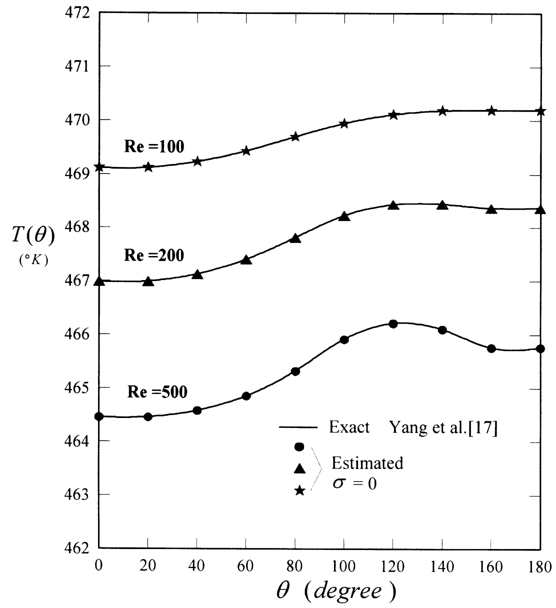

Fig. 4. The distribution of temperature along the heated cylindrical surface. ($\sigma = 0$).

methods of future time and regularization have been widely used to stabilize the results of the inverse estimation [1,3,19–20]. Those methods impose the physical condition onto the problem and increase the computational load in the estimated process. Consequently, the stability of the problem can be increased, while the computational load of the problem is also increased. In the present research, it is possible to stabilize the estimated results through a smooth process [21]. This method computes a "moving average" of the estimation. The result of data is the average of the N-point around the current point. In this process, N must be an odd number. Then, the efficiency of the estimation can be raised.

According to the above derivation, it is possible to identify whether the solution is unique or not. The method to identify the uniqueness of the solution is based on the theory of linear algebra, which will be shown in the following descriptions. If the rank of the reverse matrix **R** is less than the number of undetermined elements of the vector **C**, the number of measurements needs to be increased. Furthermore, if the rank of the reverse matrix **R** is equal to the number of undetermined elements of the vector **C**, the perpendicular distance from **C** to the column space of **E** is checked. If the distance is vanished, then the solution becomes unique.

4. Results and discussion

This paper analyzes the heat conduction and convec-

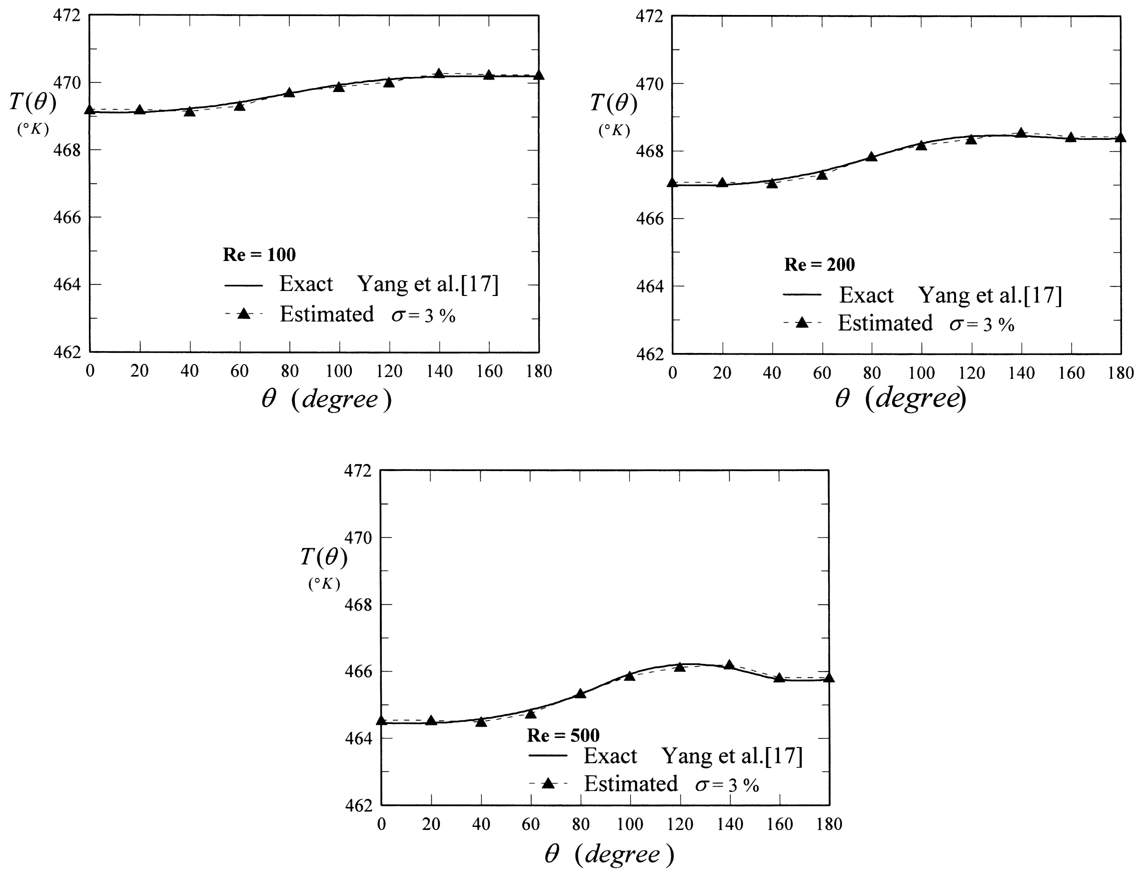

Fig. 5. The distribution of temperature along the heated cylindrical surface. (a) $\sigma = 3\%$, $Re = 100$, (b) $\sigma = 3\%$, $Re = 200$, (c) $\sigma = 3\%$, $Re = 500$.

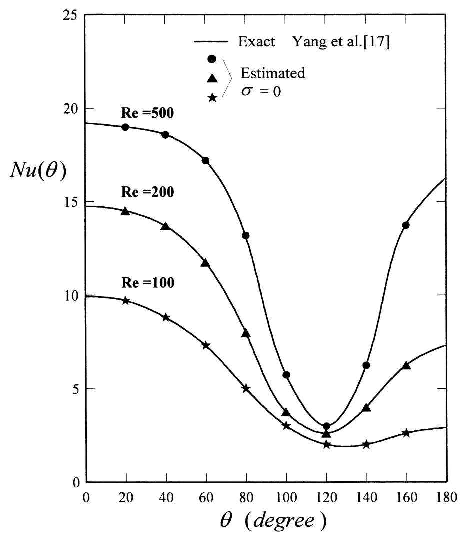

Fig. 6. The distribution of local Nusselt number along the heated cylindrical surface ($\sigma = 0$).

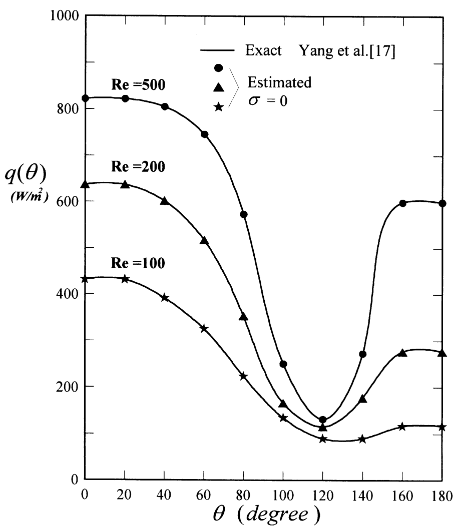

Fig. 7. The distribution of local heat flux along the heated cylindrical surface. ($\sigma = 0$).

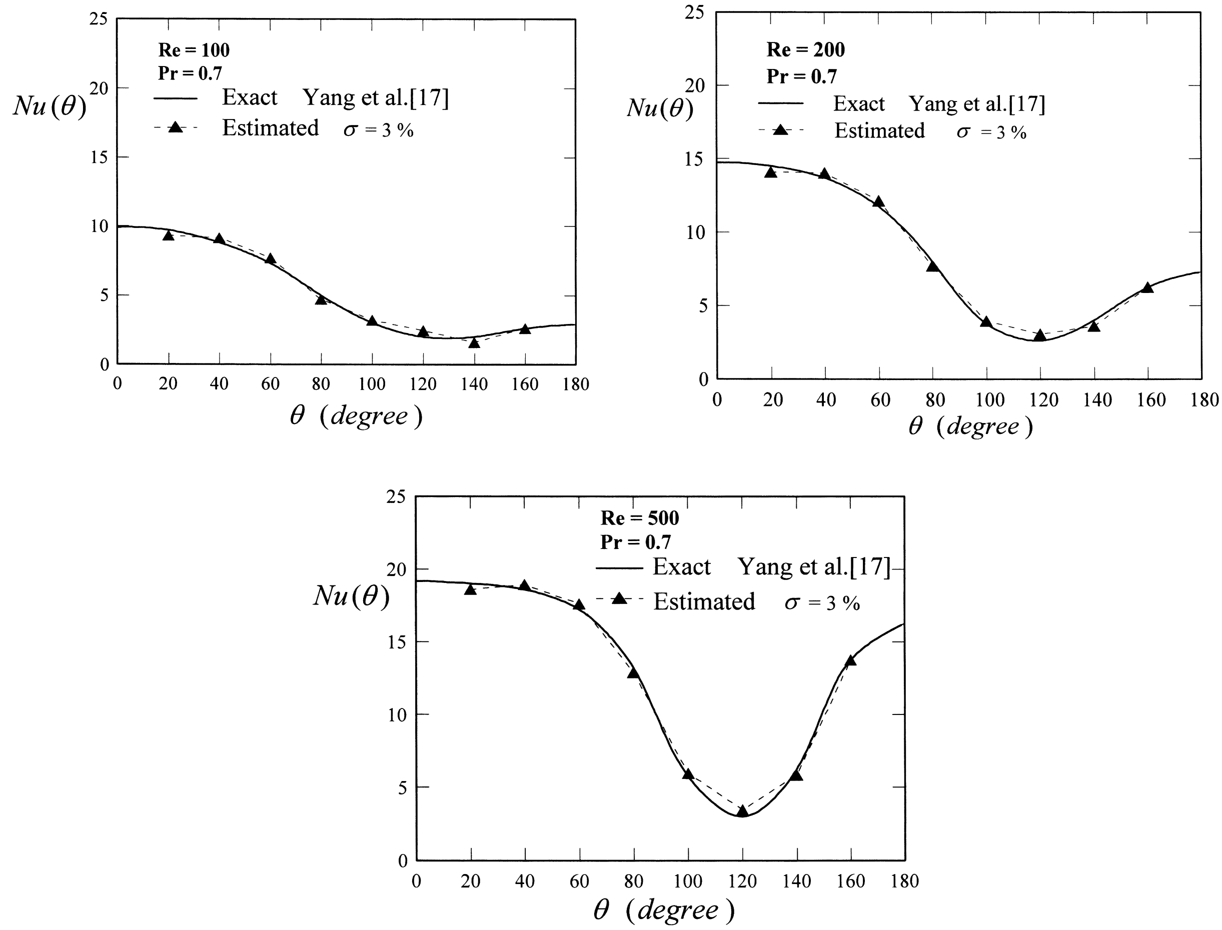

Fig. 8. The distribution of local Nusselt number along the heated cylindrical surface. (a) $\sigma = 3\%$, $Re = 100$, (b) $\sigma = 3\%$, $Re = 200$, (c) $\sigma = 3\%$, $Re = 500$.

tion problems of the cylinder normal to an air stream. Using the direct method mentioned previously combined with the boundary conditions given by Yang et al. [17], we study the Reynolds number effects on the isothermal patterns, as shown in Fig. 2. Because the hot wire is imbedded in the center of the cylinder, the heat emission from the cylindrical surface is totally caused by the forced convection of air stream. Results show that the isothermal lines near the center of the cylinder ($r = 0$) are closer than those which are far from the center. Increasing the Reynolds number tends to increase the temperature gradient. The distributions of temperature on the cylindrical surface are strongly influenced by the presence of the stagnation point, the separation point and the tail vortexes behind the cylinder. Because of the interaction between the inertia force in the flow field, the viscous effect on the cylindrical surface and the inverse pressure gradient, the flow separation occurs. Therefore, the behavior of heat transfer will be greatly influenced by the effects of the separation.

For more accurate estimation of surface conditions, the locations of the sensors are preferred closer to the cylinder surface [10]. In the present example, eight measurements are taken and the sensors are located at the eight grid points ($r = 0.04$ m, $\theta = 20n^\circ$, $n = 1$–8), which are marked in Fig. 1 with symbols $*$. The temperature data on the eight measurement locations are obtained from Fig. 2, which is calculated by the direct method, to simulate the measurements. The simulated temperature measurements used in the inverse problems are considered to include measurement errors. In other words, the random errors of simulated measurements are added to the exact temperature computed from the solution of the direct problem. Thus, the measured temperature $\mathbf{T}_{\text{measured}}$ can be expressed as

$$\mathbf{T}_{\text{measured}} = \mathbf{T}_{\text{exact}} + \omega \mathbf{T}_{\text{exact}} \quad \text{and } \omega \leq |\sigma| \tag{17}$$

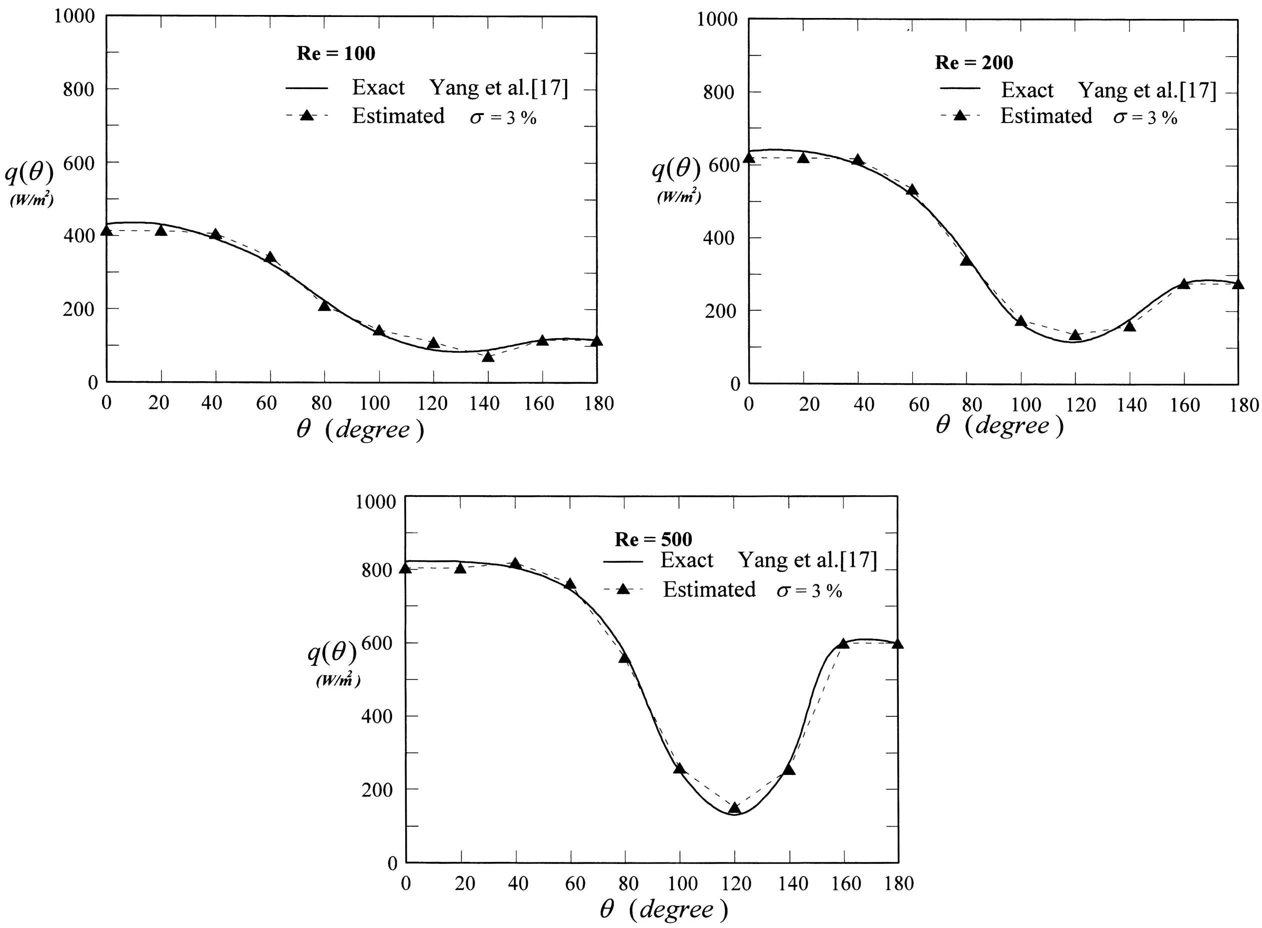

Fig. 9. The distribution of local heat flux along the heated cylindrical surface. (a) $\sigma = 3\%$, $Re = 100$, (b) $\sigma = 3\%$, $Re = 200$, (c) $\sigma = 3\%$, $Re = 500$.

where $\mathbf{T}_{\text{exact}}$ is the exact temperature, ω is the random error of the measurement, and σ is the bound of ω.

Fig. 3 shows the hot wire temperatures obtained by the inverse matrix method for $Re = 100$, 200 and 500. Results show that whether the measurement errors ($\sigma = 0$, 1 and 3%) are considered or not, the actual hot wire temperature can be predicted precisely by the proposed inverse method.

Fig. 4 shows the distributions of temperature $T(\theta)$ along the heated cylindrical surface when the measurement error is not considered ($\sigma = 0$). As θ is increased, the surface temperature increases from stagnation point ($\theta = 0°$) to the vicinity of the separation point, reaching a maximum and then reducing gradually after this point. Fig. 4 illustrates that the estimated results have excellent approximations with the results of Yang et al. [17] when measurement errors are not considered. In Fig. 5, the distributions of temperature along the heated cylindrical surface for $Re = 100$, 200 and 500 are presented. The temperature distributions obtained in the present work are in good agreement with the results of Yang et al. [17] even when the measurement error $\sigma = 3\%$ is considered.

Figs. 6 and 7 and show the distribution of the local Nusselt numbers $Nu(\theta)$ and local heat flux $q(\theta)$ along the heated cylindrical surface without measurement errors ($\sigma = 0$). Figs. 6 and 7 illustrate that the estimated results have excellent agreement with the results of Yang et al. [17], when measurement errors are not considered. The increase in the Reynolds number increases the rate of heat transfer. The estimated local Nusselt numbers and the local heat flux show a minimum at the separation point ($\theta = 120°$) because the heat transfer is weak here. The position of minimum values of $Nu(\theta)$ and $q(\theta)$ do not vary with Reynolds number. In addition, the local Nusselt numbers and the local heat flux increase at the rear of the cylinder ($\theta = 180°$) because of the mixing effect caused by flow vortexes.

The distributions of local Nusselt number $Nu(\theta)$

with $\sigma = 3\%$ for $Re = 100$, 200 and 500 are shown in Fig. 8. The minimum value of the local Nusselt numbers is located at the separation point ($\theta = 120°$). The estimated local Nusselt number distributions are in good agreement with the results of Yang et al. [17]. It indicates that the proposed method can predict this phenomenon effectively.

In Fig. 9, the distributions of local heat flux $q(\theta)$ are plotted for various value of $Re = 100$, 200 and 500 with $\sigma = 3\%$. As expected, the minimum value of the local heat flux occurs at the separation point ($\theta = 120°$). It also demonstrates that an increase of Re enhances the effect of heat transfer over the heated cylinder surface. There is a good agreement between the estimated results and the results of Yang et al. [17]; thus the proposed method is effective for the inverse heat conduction problems.

5. Conclusion

The proposed inverse method has been successfully applied to estimate the surface thermal behavior of the cylinder and the temperature of the hot wire imbedded in the center of the cylinder. An inverse formulation is reconstructed using the reverse matrix, which is derived from the governing equation and boundary conditions. The results can be solved without iteration by a linear least-squares error method. The special feature of the proposed method is that the uniqueness of the solution can be identified. The present study of the heated cylinder normal to a laminar air stream has been used to evaluate the accuracy and the robustness of the proposed method. From the results, it appears that by using the proposed method, without measurement error, the exact solution can be found even with only few measuring points. When measurement errors are included, in order to enhance stability and accuracy, temperature data requires more measuring points at locations inside the cylinder.

This proposed inverse method requires no prior information on the functional form of the unknown quantities, no initial guesses, and no iterations in the calculating process. Furthermore, the uniqueness of the solutions can easily be identified. This implies that the present model offers a great deal of flexibility. Through the proposed method, the surface and central thermal behavior can be obtained merely by the inexpensive measurement such as infrared measuring devices or thermocouples. Thus, expensive sensors for the direct measurement are not needed any more and the difficulties encountered in the measuring processes can be avoided. Consequently, the results confirm that the proposed method is effective and efficient for two-dimensional inverse heat conduction problems.

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